Correlation Effects on the Size of the Θ^+ Pentaquark

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Abstract

The root mean square radius of the Θ^+ pentaquark is investigated based on a number of different quark cluster models both analytically and numerically. An oscillator wavefunction with a simple Gaussian form is used to produce a range of results for the uncorrelated potential in the COSMA, Lipkin/Karliner and Jaffe/Wilczek models of $< R^2 >^{\frac{1}{2}}$=0.55-0.77, 0.87-1.06 and 0.86-1.05fm respectively. The FaCE tool is used to provide the numerical solution to the Jaffe/Wilczek model for a more realistic correlation between the cluster potentials, and produces an rms radius range of 0.99-1.06fm.

Contents

1 Introduction

At their most elementary levels all matter is composed of fundamental particles, quarks and electrons. The quarks combine and interact via gluons, the mediators of the strong force. This interaction is responsible for holding nuclei together and allowing fusion to occur in stars, hence allowing the production of heavy nuclei and life itself. Quantum chromodynamics (QCD) is the underlying theory of the strong force and is a quantum field theory that successfully embodies both special relativity and quantum mechanics [1]. However, its equations are complicated, making it difficult to calculate the masses and properties of familiar particles from first principles. There are, however, various simplifications and models which are either QCD-based or incorporate its main principles to explain the spectrum of hadrons observed. Perhaps the most successful is the quark model, which is based on the principle that all hadrons are either composed of quark anti-quark pairs (mesons) or three quarks (baryons) [2]. There are also six quark flavours; up, down, strange, charm, top and bottom, which can have one of three colour states; red, green or blue, combining to make a 'neutral' (or 'colourless') state. The type of meson or baryon depends on which quark flavours the particle consists of. Any state with a quark content other than this is beyond the quark model and termed as exotic, although the existence of such exotic states have been proposed since the early days of QCD.

One of the most exciting developments in particle physics over the past few years has been the experimental evidence for the existence of a particle consisting of five quarks, named the pentaquark, Θ^+ [3]. Since then, the existence of other similar exotic states have been claimed by many labs around the world, but as yet none have been conclusively identified. While the jury is still out on the existence of the Θ^+ and other states as real particles, theoretical work is necessary for understanding their fundamental properties and the way in which they form. This work could provide exciting new insights into the subtleties of the strong interaction.

The aim of this project is to investigate the size, or root mean square radius of the Θ^+ pentaquark based on a number of different model calculations, both analytically and numerically. This five-body problem is simplified according to three quark cluster models, which take into account the relative orbital angular momentum of the clusters and the types of potential between them. Initially, the wavefunctions for each model are solved for a situation where the potentials between each cluster are independent of each other. It is then possible to further study one of these models by modifying an existing piece of Fortan code called FaCE. This is a three-body code, used in nuclear theory, particularly in the study of halo nuclei and is useful as it provides a much more realistic correlated potential between the quark clusters. The results from each model are compared and the factors which appear to affect the size of the Θ^+ discussed.

2 Background Theory

It was originally thought that particles containing more than three quarks would not exist. This is due to the fact they should be very unstable and decay into two lighter particles so quickly that the states would effectively be unobservable. However, pentaquarks have since been predicted through QCD, and there are recent claims of the observation of the Θ^+ pentaquark with a finite lifetime. The Θ^+ decay process into a proton (uud) and kaon (u$\bar{\text{s}}$) provides evidence for the minimum composition of uudd$\bar{\text{s}}$, with one unit of positive charge, positive strangeness and a baryon number of one [4]. Its mass is found to be 1540MeV, although other properties of the particle including angular momentum and parity remain undetermined. One of the major issues of the Θ^+ which creates a challenge in terms of understanding its internal structure is that the claimed lifetime turns out to be a lot larger than expected. The lifetime of a particle is generally thought of in terms of its width, or spread of rest mass energy. The larger the width, the shorter the lifetime. The Θ^+ pentaquark has a width measuring $\sim 20MeV$, which is an order of magnitude (or 100 times) lower than what is expected of a conventional baryon decaying via the strong interaction [5]. It is thought that the extra binding required to give this particle its long lifetime could be due to the quarks forming clusters inside the Θ^+. Among the many theoretical models for the exotic particle are the three-cluster state proposed by Robert Jaffe and Frank Wilczek [6] and the diquark-triquark model put forward by Harry Lipkin and Marek Karliner [7]. These two models form the basis for the investigation into the size of the Θ^+.

In order to calculate the size, or root mean square radius of the pentaquark, it is necessary to consider the interaction between each of the quarks and clusters in the particle. The potentials between each of the quarks is written in terms of centre of mass and relative coordinates, or in terms of Jacobi coordinates for the three-body problem, then the Schrodinger equation is solved to obtain the wavefunction of the particle, where the energy eigenvalue is its rest mass energy. The five-body system of the Θ^+ is however much too complicated to solve exactly in this way, so further simplifications or complex computer codes are required. The simplest starting point is to assume that the potentials between each of the clusters are independent from each other (i.e. decoupled). Although not very realistic, this allows the use of simple analytical formulae for the radii in each of the pentaquark models.

2.1 The Single Particle/COSMA Model

The single particle model is the simplest way to model the pentaquark, as it assumes all the constituent quarks are independent of each other. It is analogous to the Cluster Orbital Shell Model Approximation (COSMA), which is commonly used in nuclear theory. It assumes that the overall wavefunction of the particle is the product of the individual wavefunctions for each of the constituent particles with respect to one of them. In this case the position of the up and down quarks are defined relative to the anti-strange (Figure 1).

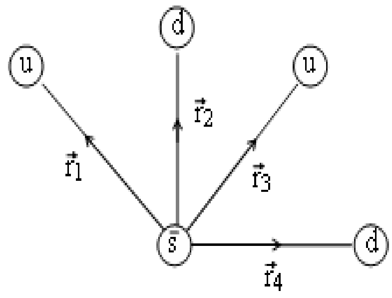

Figure 1: Quark arrangement in the COSMA Model

The wavefunction is then written as a product of uncorrelated single particle states:

$$\Psi = \phi(\vec{r_1})\phi(\vec{r_2})\phi(\vec{r_3})\phi(\vec{r_4}) \tag{1}$$

Each pair of quarks are then assumed to have oscillator wavefunctions in relative s-waves. This means that the angular momentum between each pair is L=0. Each of the quarks can be in this state according to the Pauli Exclusion Principle if each of the two like-quarks have opposing spin. This type of s-wave oscillator is given by a simple Gaussian function, where α is the 'oscillator parameter', which defines how tightly each quark is bound to the $\bar{\text{s}}$:

$$\phi = A \exp[-\frac{r^2}{2\alpha^2}] \tag{2}$$

The constant A is found by normalisation, where:

$$\int_0^\infty \mid \phi \mid^2 \mathrm{d}\vec{r} = 1 \tag{3}$$

Remembering that r is a radial distance in terms of spherical polar coordinates:

$$\int_0^\infty A^2 \exp[-\frac{r^2}{\alpha^2}] 4\pi r^2 \mathrm{d}r = 1 \tag{4}$$

Using the standard integral:

$$\int_0^\infty r^{2n} \exp[-ar^2]\mathrm{d}r = \frac{(2n-1)!!}{2(2a)^n}\sqrt{\frac{\pi}{a}} \tag{5}$$

A is found to be:

$$A^2 = \frac{1}{\pi\sqrt{\pi}\alpha^3} \tag{6}$$

The mean square radius of the pentaquark for this method is defined as [8]:

$$< r^2 >= \frac{4m}{M}(1 - \frac{m}{M}) < \Psi \mid r_1^2 \mid \Psi > \tag{7}$$

where: m = Mass of u and d quark

M = Mass of pentaquark

This formula simplifies further as:

$$< \Psi \mid r_1^2 \mid \Psi >= \int \mid \phi(r_1) \mid^2 r_1^2 dr_1 \int \mid \phi(r_2) \mid^2 dr_2 \int \mid \phi(r_3) \mid^2 dr_3 \int \mid \phi(r_4) \mid^2 dr_4 \tag{8}$$

Most of these integrals reduce to one as they satisfy the normalisation condition, Eq 3, Therefore:

$$< \Psi \mid r_1^2 \mid \Psi >= \int_0^\infty A^2 \exp[-\frac{r^2}{\alpha^2}] r_1^2 dr_1 \tag{9}$$

Since the mass of the u and d quarks are approximately the same, it can be assumed that the potential between each of the four quarks with the $\bar{\text{s}}$ are also the same, so it does not matter which of the vectors we define as r_1, since all of the oscillator parameters will have the same value. This equation can then be solved using a standard integral, Eq 5, once more to obtain a formula for the mean square radius and hence the rms value:

$$< r_1^2 >= \frac{4m}{M}(1 - \frac{m}{M})(3/2\alpha^2) \tag{10}$$

2.2 The Karliner/Lipkin Model

This model suggests the structure for the pentaquark is a diquark coupled to a triquark in a relative p-wave (Figure 2) [7]. Karliner and Lipkin predict this state to have a spin parity of $J^p = 1/2^+$ and isospin, I=0, which produces a mass that is close to the experimental value. Although the parity of the particle has yet to be determined experimentally, they suggest that its observed narrow width most likely comes from a relative p-wave structure, meaning that there is one unit of angular momentum, L=1, between the diquark and triquark clusters, ensuring that the overall Θ^+ parity is positive.

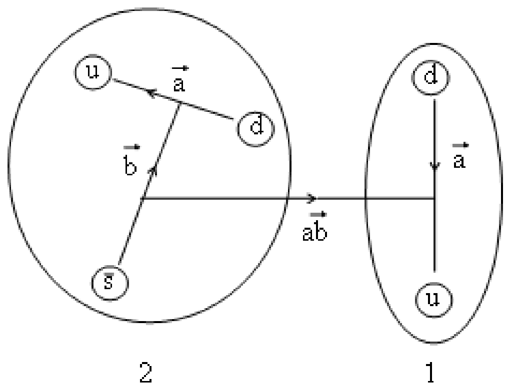

Figure 2: Quark arrangement in the Karliner/Lipkin Model

The overall radius of the particle for this model is then given simply by the radius of each of the two clusters, and the separation between them, with some weighting factor of the masses, where m_1 and m_2 are the masses of each cluster:

$$M < R^2 >= m_1 < r_1^2 > +m_2 < r_2^2 > + \frac{m_1 m_2}{M} < r_{12}^2 > \tag{11}$$

However, in this case, cluster 2 can be classed as two separate clusters in itself, as a ud coupled to an $\bar{s}$, so Eq 11 must first be used to calculate its radius. However, the $\bar{s}$ quark is point-like, so it has a radius of zero and the term disappears from the equation. The vectors involved in each of the clusters will depend on the oscillator parameter in the same way as in the COSMA model for relative s-waves, so the $\frac{3}{2}\alpha^2$

dependence can be put back in. However, if considering the relative p-wave state, the oscillator wavefunction will be different:

$$\phi = Ar\exp[-\frac{r^2}{\alpha^2}] \quad (12)$$

The normalisation condition can be applied once again, and the standard integral, Eq 5, solved in the same way as before to obtain a $\frac{5}{4}\alpha^2$ dependence on the cluster separation. Since there is only angular momentum between the two main clusters, the α dependence within each cluster remains as $\frac{3}{2}$, for an s-wave.

2.3 The Jaffe/Wilczek Model

Jaffe and Wilczek suggest that the Θ^+ can be considered as a bound state of an anti-quark with two highly correlated spin-zero diquarks [6]. In this model, the lighter the quarks, the stronger the correlation, which helps the formation of the two diquarks, where they obey Bose statistics. They propose a state with overall positive parity, where only the s-wave configuration is likely (Figure 3).

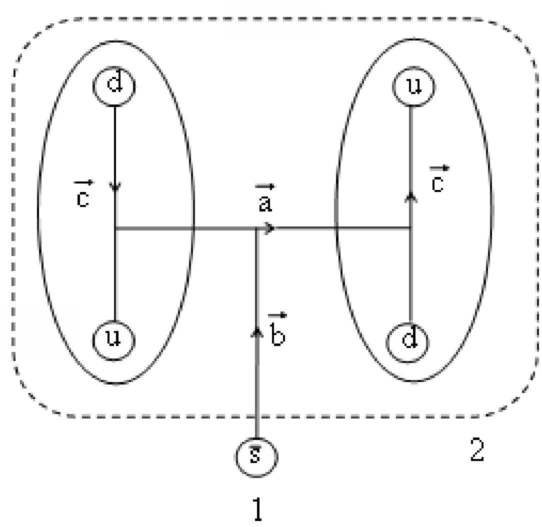

Figure 3: Quark arrangement in the Jaffe/Wilczek Model

Eq 11 can be used to calculate the rms radius in a similar way to the Karliner/Lipkin model by considering the radius of the two diquarks, the separation between them, and the separation between the s̄ and pair of diquark clusters. Once again the s-wave Gaussian oscillator wavefunction can be used for each of the clusters, so that the oscillator parameter simply needs to be chosen in each case.

2.4 The FaCE Model

Since the Jaffe/Wilczek model represents the pentaquark as a three-body problem, it is possible to further investigate it using a tool called FaCE [9](which stands for Faddeev Calculations with Core Excitation). FaCE is a Fortran programme, which was originally designed to numerically solve three-body problems in nuclear physics, particularly halo nuclei. So far, the methods discussed for dealing with the pentaquark have assumed completely independent potentials between the quarks and clusters in each model, however in a more realistic situation, the separation of each of the clusters will depend on the potentials between all of the constituents. The FaCE tool calculates energies and eigenvalues, including the rms radius for nuclei by solving the three-body Faddeev equations. This is modified to allow the option of simple quark-quark potentials in the Θ^+ particle. An input file containing the relevant information about the pentaquark can be read in by the programme and an investigation into the size carried out, by selecting a relevant potential between the quarks.

3 Procedure

The starting point for this investigation is to use the COSMA model for the pentaquark to provide an estimate of the root mean square (rms) radius. In this representation each of the four u and d quarks are assumed to interact in the same way with the s̄. The masses chosen for each of the quarks are $m_{u,d} = 360MeV$, $m_s = 540MeV$ and $M_{\Theta^+} = 4m_{u,d} + m_s = 1980MeV$ respectively [7]. The value for the oscillator parameter then needs to be chosen, and this is done by taking a typical range of hadronic interactions as $\sim 1fm$ [10]. The value for α is then calculated from the relationship for an s-wave oscillator:

$$< R^2 >= \frac{3}{2}\alpha^2 = 1 \Rightarrow \alpha = \sqrt{\frac{2}{3}} \qquad (13)$$

This value for α can then be used in Eq 10 to provide a first estimate.

A similar procedure can be carried out for the Lipkin/Karliner model in an s-wave. However, due to the clustering of the quarks in this case, it can be assumed that there is stronger binding within the clusters than there is between them, so the oscillator parameter will be different in each case. If the cluster separation is held constant at 1fm, the separation of the quark pairs in each of the clusters can be varied to investigate the effect on the Θ^+ size. Since Lipkin and Karliner suggest a relative p-wave configuration between the clusters, the oscillator parameter for the separation of the diquark and triquark can be adjusted accordingly to see how the angular momentum affects the size. By then varying the separation of the u and d pairs in the two diquarks of the Jaffe/Wilczek model, it is possible to determine whether the way the quarks are clustered makes a great difference to the radius of the pentaquark.

The FaCE tool is a way of seeing how a more realistic potential will affect the pentaquark size, and it is set up so that an input file containing all the necessary parameters are read in by the programme. Information about the mass, charge, spin and parity of each cluster is required, along with the total parity of the particle. The type of potential between the clusters is also specified by choosing one of the options set up in FaCE. Initially, a simple confinement potential is used, with the linear form, where V_0 and C are the confinement parameters:

$$V_{conf} = V_0 + Cr \qquad (14)$$

A first estimate for these two free confinement parameters is taken from the results of Glozman *et al.*[11], where they were determined from a fit to the baryon spectra in their chiral constituent-quark model. These values were however chosen out of a possible set that lead to a similar description of baryon spectra, so it is possible to adjust them in an attempt to produce a sensible set of values for the ground state energy and rms radius in the output for the pentaquark.

The next step is to use a more realistic potential between the quarks. Such a potential is the linear harmonic oscillator, previously used for the analytical solution of the pentaquark models. This type of potential is used for the study of particles with mass m, which are attracted to a fixed centre by a force proportional to the displacement from the centre [12], and has the form:

$$V = \frac{1}{2}kr^2 \tag{15}$$

k is the force constant, and is defined in terms of the angular frequency, ω, and reduced mass, μ:

$$\omega = \sqrt{\frac{k}{\mu}} \tag{16}$$

This potential is then included in the Hamiltonian operator, where the Schrodinger energy eigenvalue equation has the solution:

$$\Psi(r) = (\frac{1}{a\sqrt{\pi}})^{\frac{3}{2}} \exp[-\frac{r^2}{2a^2}] \tag{17}$$

The parameter a has the units fm, and is defined as:

$$a = \sqrt{\frac{\hbar}{\mu\omega}} \tag{18}$$

A substitution is then made for ω and the equation rearranged to define k in terms of quantities that are known, or can be calculated.

$$k = \frac{(\hbar c)^2}{\mu c^2 a^4} \tag{19}$$

Values of k can then be specified in the pentaquark input file, by calculating the reduced mass for each cluster and choosing values for the parameter a. These values can be chosen based on the parameter used in the analytical solution of the Jaffe/Wilczek to investigate how the correlated potentials of the FaCE model affects the results.

There were many issues to be taken into consideration during this investigation and a variety of unknowns which could have affected the outcome of the results. The first major issue was the masses of the quarks used throughout the calculations. Due to the fact that free quarks have never been observed, with QCD theory confining them in hadrons, never to escape, it is impossible to know the exact mass of a single quark, and as a consequence there is some distribution in the values quoted. The mass of the Θ^+ that has been used throughout is 1980MeV, although the observed value is quoted as 1540MeV. The decision to use the higher mass (the sum of the individual quark constituents) was taken due to the fact that it is not known how the binding energy is distributed within the clusters for the models used. Rather than try to estimate the masses of the separate clusters by accounting for some binding energy, it was more sensible to be consistent and simply exclude this factor, which will obviously affect the result in some way.

In terms of using the FaCE programme, various information about the Θ^+ was required in the input file, such as the spins, masses and parities of each cluster and the particle as a whole. Although some of this information has been experimentally determined by the original observations of the claimed pentaquark, much of it remains undetermined. Where possible, the predictions of Jaffe and Wilczek have been used [6].

It should also be noted that while a number of research groups have confirmed the original findings of the Θ^+, many other experimental groups have claimed negative results [13].

4 Results

Using Eq 10, calculated previously and the values stated for the quark masses, the rms radius of the Θ^+ for the COSMA model is found to be:

$$< R^2 >^{\frac{1}{2}} = \sqrt{0.893\alpha^2} \tag{20}$$

A range for the radius is then estimated by choosing values of α^2, which correspond to u-$\bar{s}$ separations of 0.5-1fm. This range is found to be 0.55-0.77fm and is represented in Table 1:

u-d Separation (fm)	α^2	$< R^2 >^{\frac{1}{2}}$ (fm)
0.5	0.33	0.55
0.6	0.40	0.60
0.7	0.47	0.65
0.8	0.53	0.69
0.9	0.60	0.73
1.0	0.67	0.77

Table 1: Results from the COSMA Model calculations

The quark and cluster masses are included in Eq 11 to provide a formula for the radius based on the Lipkin/Karliner model in a relative s-wave:

$$< R^2 >^{\frac{1}{2}} = \sqrt{1.09\alpha_a^2 + 0.23\alpha_b^2 + 0.35\alpha_{ab}^2} \tag{21}$$

With the cluster separation held constant at 1fm, the oscillator parameter becomes $\alpha^2 = \frac{2}{3}$ for a relative s-wave. The parameters between the quarks in the clusters can be varied to obtain a range for $< R^2 >^{\frac{1}{2}}$. The ud-$\bar{s}$ separation is initially held constant, also at 1fm and the u-d separation is varied to obtain an upper range estimate for the rms radius. This range is found to be 0.87-1.06fm (Table 2).

It would generally be expected that since the $\bar{s}$ is bound in cluster 2 (Figure 2), it will experience a stronger potential than the two clusters themselves. If this is the case, then the oscillator parameter, α_b, should be different from that of the cluster separation parameter, α_{ab}. Based on the principle that the lighter the quark, the tighter they are bound, the lightest u-d pairs are now held constant at a separation of 0.5fm each and the separation of the ud-$\bar{s}$ varied between 0.5-1fm, keeping the cluster separation at 1fm ($\alpha_{ab}^2 = \frac{2}{3}$). This produces a slightly lower range for the Θ^+ radius of 0.82-0.87fm.

The p-wave configuration for this model indicates that there is one unit of angular momentum between the two clusters. This means that rather than just having a simple Gaussian for the oscillator wavefunction between the diquark and triquark, it will have the form $r \times Gaussian$, giving a $\frac{5}{2}\alpha^2$ dependence on the cluster separation (i.e. $\alpha^2 = \frac{2}{5}$ for a 1fm separation). The formula for the rms radius is therefore modified slightly from the s-wave calculation:

$$< R^2 >^{\frac{1}{2}} = \sqrt{1.09\alpha_a^2 + 0.23\alpha_b^2 + 0.58\alpha_{ab}^2} \tag{22}$$

However, since the cluster separation term is held constant at 1fm, the α dependence is normalised to 1, (by multiplying by 2/5, or 2/3 for th s-wave case). This leaves just the mass ratio, i.e. a constant, so the range of $< R^2 >^{\frac{1}{2}}$ is not affected.

u-d Separation (fm)	α_a^2	$< R^2 >^{\frac{1}{2}}$ (fm)
0.5	0.33	0.87
0.6	0.40	0.91
0.7	0.47	0.95
0.8	0.53	0.98
0.9	0.60	1.02
1.0	0.67	1.06

Table 2: Results from the Lipkin/Karliner Model calculations

An s-wave configuration using the Jaffe/Wilczek model produces the following relationship for the rms radius of the Θ^+:

$$< R^2 >^{\frac{1}{2}} = \sqrt{1.09\alpha_c^2 + 0.27\alpha_a^2 + 0.30\alpha_b^2} \qquad (23)$$

The separation of the two diquarks, and that of the $\bar{\text{s}}$ to the diquark pair is held constant at 1fm, while the separation of the u and d in each of the pairs is varied to produce a range of $< R^2 >^{\frac{1}{2}}$=0.86-1.05fm (Table 3):

u-d Separation (fm)	α_c^2	$< R^2 >^{\frac{1}{2}}$ (fm)
0.5	0.33	0.86
0.6	0.40	0.90
0.7	0.47	0.94
0.8	0.53	0.98
0.9	0.60	1.02
1.0	0.67	1.05

Table 3: Results from the Jaffe/Wilczek Model calculations

The input file for the FaCE programme requires information about each of the clusters in the three-body system (the Jaffe/Wilczek model in this case). Each of the three clusters in this model have a charge of $\frac{1}{3}$e, but the two u-d clusters will have a finite size, whereas the $\bar{\text{s}}$ quark is classed as a point-like object with zero size. The chosen parity of the Θ^+ is positive, to be consistent with Jaffe and Wilczek's theories and initially the potential between each cluster is specified as a confinement potential, Eq 14. Values of $V_0 = 416.0$ and $C = 2.33$ produce a ground state energy of $-1181MeV$, and rms radius of 1.65fm. The energy of the ground state is the equivalent of the binding energy of the particle, and the negative result suggests a bound state, so a value of $\sim -440MeV$ should be expected. The rms radius is also a little large compared to the values obtained from the three methods discussed previously. Further adjustments to the fitting parameters were made in order to produce an energy closer to the expected value and a radius closer to that of the previous results, with values of $V_0 = 320$ and $C = 100$ producing an energy of $-442MeV$ and rms radius of 1.11fm.

The oscillator potential, Eq 15, is then used in attempt to produce the most realistic correlated model for the pentaquark. The k parameter, which is included in the input file can be calculated for each of the clusters using Eq 19, where $\hbar c$ is quoted as 197.3MeV [12]. The reduced mass (μ), is calculated for each diquark as 360MeV and μ for the ud-$\bar{s}$ interaction as 308MeV. If the parameter a is fixed at 1fm, an energy of -441MeV and rms radius of 1.06fm is obtained. This result is incredibly close to the expected binding energy value, and produces a radius well within the range of values calculated from the previous model calculations. Varying k for a range of u-d separations between 0.5-1fm, in a similar way to the analytical Jaffe/Wilczek calculations produces a range of rms radii of 0.99-1.06fm. The range of ground state energies in this case is found to be 898-441MeV. These results are represented in Table 4 below:

u-d Separation (fm)	k	$< R^2 >^{\frac{1}{2}}$ (fm)	Energy, E (MeV)
0.5	1728	0.99	-898
0.6	833	1.00	-705
0.7	450	1.02	-591
0.8	264	1.04	-519
0.9	165	1.05	-473
1.0	108	1.06	-441

Table 4: Results from the FaCE calculations in an oscillator potential

5 Conclusion

The various models and calculations used in this project have produced a range of possible values for the rms radius of the Θ^+ pentaquark. As a first estimate, the uncorrelated COSMA model produced results between $< R^2 >^{\frac{1}{2}}$=0.55-0.77fm for a u-d separation of 0.5-1fm. Similar separations were investigated for the Lipkin/Karliner, Jaffe/Wilczek and FaCE models to produce a set of results that were around 40% larger than the COSMA predictions. The Lipkin/Karliner calculation produced a range of $< R^2 >^{\frac{1}{2}}$ that was within 1% of the results from the Jaffe/Wilczek model, with values of 0.86-1.05fm and 0.87-1.06fm respectively. The FaCE model using the oscillator potential produced a set of rms radii of 0.99-1.06fm. This result can be directly compared to the analytical solution for the Jaffe/Wilczek model to gain some insight into the effect that the correlated potentials have on the size of the Θ^+. At smaller u-d separations, the FaCE model predicts a larger rms radius than the Jaffe/Wilczek calculation, with $< R^2 >^{\frac{1}{2}}_{FaCE}$ being 14% larger at a separation of 0.5fm. This is a significant difference on the particle scale. As the u-d separation increases, FaCE predicts a slower increase in the rms radius, with both models predicting a value of 1.06fm at a 1fm u-d separation. The slower increase is expected, because as the separation of one cluster changes, the others will also adjust in the correlated potential. Finally, a further indication for the size of the pentaquark is the energy values that are output by the FaCE model. Since the mass of the constituent quarks is 1980MeV, but the observed mass of the Θ^+ is around 1540MeV, approximately 440MeV must be used up as binding energy. FaCE predicts energies which are very close to this value for the higher u-d separations, which could provide validation for the calculated radii.

If correlated quark potentials are included in the three-body Jaffe/Wilczek model the size of the Θ^+ pentaquark is predicted to be larger than in the uncorrelated case, provided each (ud) diquark is tightly bound. However, the correlation has little effect if the (ud) diquarks are weakly bound.

Although the existence of the Θ^+ pentaquark is still a debated topic [14], theoretical work on this particle and other exotic states is necessary for a greater understanding of the strong interaction.

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